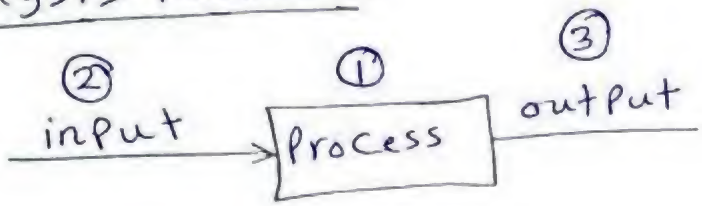
م. شهاب

Control Lec 8

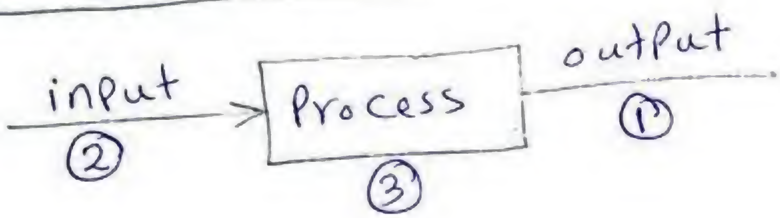
"Control system Design"

Engineerng Problems

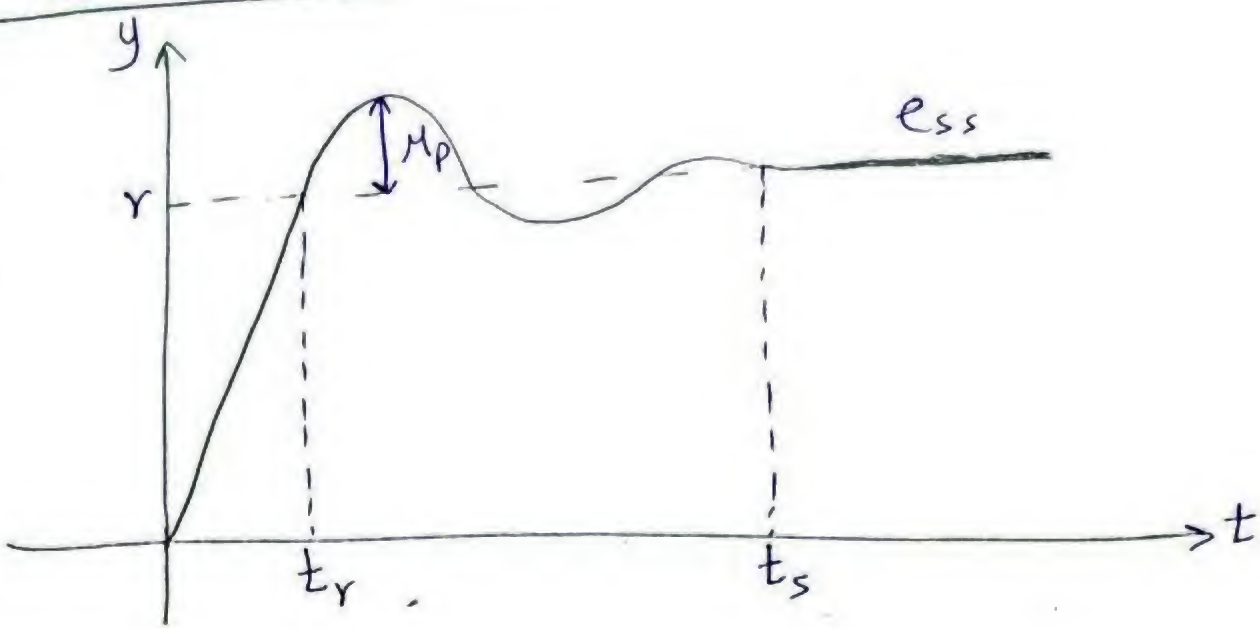
→ Analysis Problem:-



→ Design problem:-



Control systems Design specs:-



$$t_r = \frac{\pi - \cos^{-1} \zeta}{\omega_n \sqrt{1-\zeta^2}} \quad ; \quad t_p = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}}$$

$$M_p = e^{-\zeta \pi / \sqrt{1-\zeta^2}} \quad ; \quad t_s = \frac{4}{\zeta \omega_n}$$

e_{ss} : steady state error

type 0 : $e_{ss} = \dfrac{1}{1+K_p}$

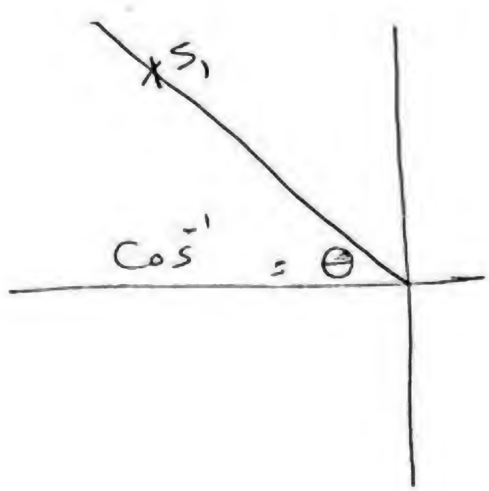
type 1 : $e_{ss} = \dfrac{1}{K_v}$; type 2 : $e_{ss} = \dfrac{1}{K_a}$

$$K_p = \lim GH(s) \quad ; \quad K_v = \lim_{s \to 0} s\,GH(s)$$

$$K_a = \lim_{s \to 0} s^2\,GH(s)$$

* Design Procedure

1. desired C.L. c/c eqn



$$s_{1,2} = \underbrace{-\zeta \omega_n}_{\sigma} \pm \underbrace{\omega_n \sqrt{1-\zeta^2}}_{\omega_d}$$

- Control Design objective :-

Add dynamic components to the system to change the overall system dynamics i.e. change the locations of C.L. poles.

Remember what we've done in Practice :-

* Requirements
* Modelling of system dynamics (Analysis)
* Controller (Add Components)

Response

Transient — M_p, t_r, t_s

steady state — e_{ss}

* Transient Response Compensation

→ lead Compensator. → PD

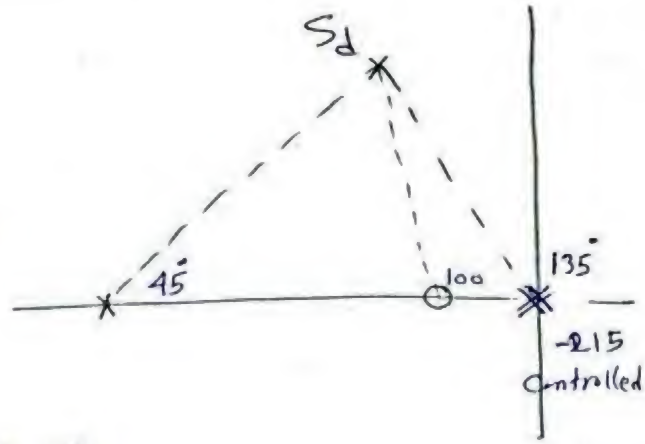
* steady state error Compensation

→ lag Compensator. → PI.

* for overall Compensation

→ lead-lag Compensator. → PID.

* Lead Compensator



at

$s_d : \sum \phi_z - \sum \phi_p \neq \pm 180$

desired

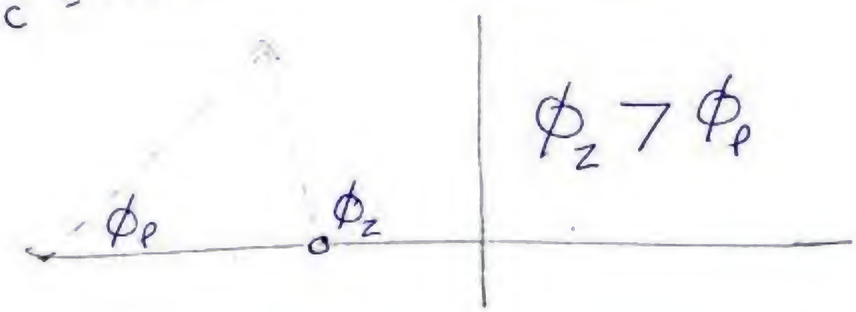
at $s_d : \sum \phi_z - \phi_p = \pm 180$

By adding zero & pole to the system

$$\sum \phi_{z_c} - \sum \phi_{p_c} = \pm 180$$

$$\angle GH(s) + \phi_c = -180$$

Root locus Angle Condition



$\phi_z > \phi_p$

* lag Compensator design

① Calculate $\beta = \frac{K_c}{K_{ur}} * 1.1$ → safety factor

K_{ur} → uncontrolled

K_c: desired value of dc gain.

K_{un}: the system value of Dc gain.

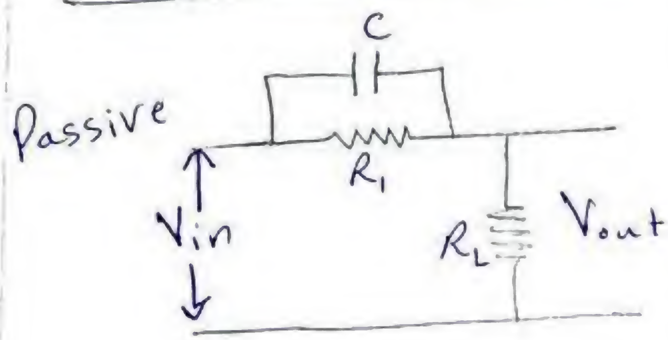
② Assume zero location (Z_c) in the range of 10% from the 2nd dominent pole of the sys.

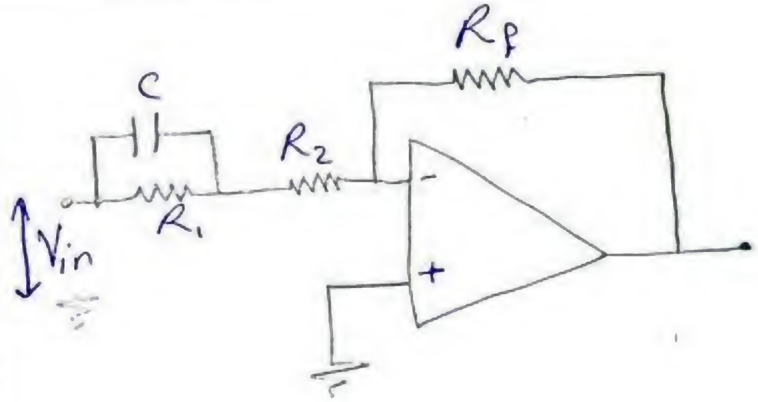
$$Z_c = \frac{10}{100} * P_2 \qquad P_2 \Rightarrow \text{2nd dominent pole}$$

$$P_c = \frac{Z_c}{\beta} \qquad \underset{\text{of Controller}}{K_{dc}} = \frac{Z_c}{Z_c/\beta} = \beta$$

* lead Compensator

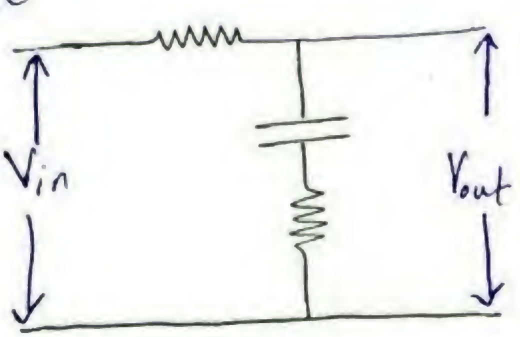
Passive

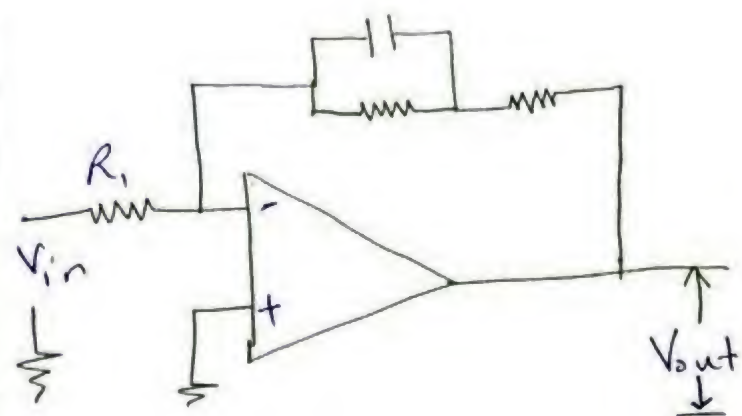




* lag Compensator

Passive





Example

A type 1 system has O.L.T.F

$$GH(s) = \frac{K}{s(s+1)(s+4)}$$; It is desired to Compensate such system to match the following design specs :-

$\zeta = \frac{1}{2}$ $\qquad e_{ss} \leq 0.1$

$\omega_n = 2$ rad /sec

$\boxed{\text{solution}}$

① desired locations for closed

$$S_{1,2} = -\zeta\omega_n \pm \omega_n\sqrt{1-\zeta^2}$$

$$= -1 \pm J\sqrt{3}$$

② from the angle Condition

$$\angle GH(s) + \phi_c = -180$$

$(-1+J\sqrt{3}+1)$

$$\angle \frac{K}{s(s+1)(s+4)} = 0 - \angle -1+J\sqrt{3} - \angle J\sqrt{3} - \angle 3+J\sqrt{3}$$

$\Downarrow$

$GH(s)$

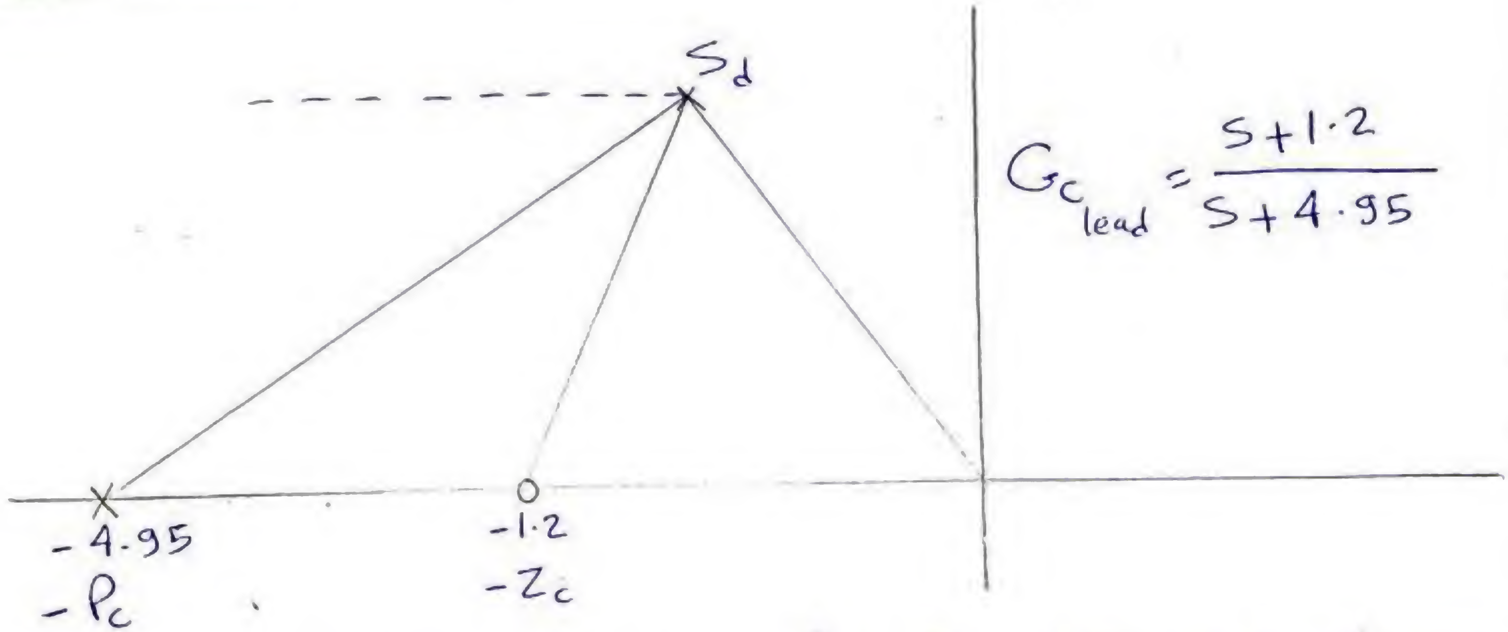
$\overset{s}{=}$

$(-1+J\sqrt{3}+4)$

$\boxed{6}$

$$-120 - 90 - 30 + \phi_c = -180 \Rightarrow \boxed{\phi_c = 60^\circ}$$

③ Determine the locations of Z_c & P_c



$$G_{c_{lead}} = \frac{S+1.2}{S+4.95}$$

← طريقة تحديد مكان Z_c ، P_c ← مذكورة في محاضرة رقم 7 .

The total T.F

$$GH(s) = \frac{K(S+1.2)}{s(S+1)(S+4)(S+4.95)}$$

④ Determine K from magnitude Condition

$$\|KGH(s)\| = 1$$

$$K = \frac{1}{\| GH(s) \|} = \left. \frac{\|s\| \, \|s+1\| \, \|s+4\| \, \|s+4.95\|}{\|s+1.2\|} \right|_{s=-1+j\sqrt{3}}$$

$K = 30$

← داخل المقياس نعوض $(s=-1+\sqrt{3})$
ونحسب بالآلة الحاسبة بعدا.

⑤ check steady state error

type1 system $\Rightarrow e_{ss} = \frac{1}{K_v}$

$$K_v = \lim_{s \to 0} s\,GH(s)$$

$$K_v = \lim \frac{s(s+1.2) * 30}{s(s+1)(s+4)(s+4.95)} = 1.81$$

$e_{ss} = \frac{1}{K_v} = 0.55$, $\underbrace{e_{ss} \leq 0.1}_{\text{required}}$

$\Rightarrow$ to reduce e_{ss} Add lag Compensator

desired $e_{ss} = 0.1 \Rightarrow K_v = \frac{1}{e_{ss}} = 10$

* lag Compensator design

① $K_{un} = 1.82$, $\underbrace{K_c = 10}_{\text{Controlled}}$

$\beta = \frac{10}{1.82} * \underbrace{1.1}_{\text{safety factor}} = 6.04$

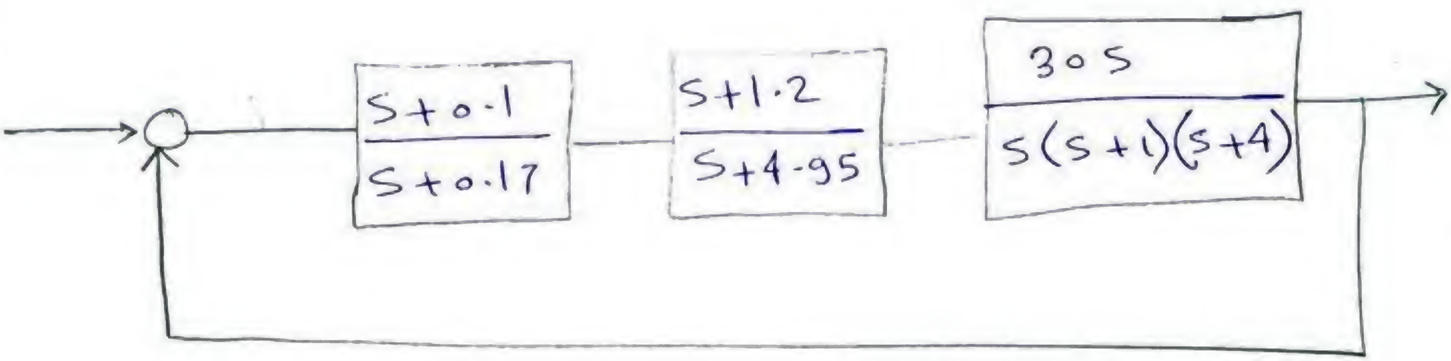
② Locations of Z_c and P_c

$$Z_c = \frac{10}{100} * P_2 = 0.1 * -1 = -0.1$$

$$P_c = \frac{Z_c}{\beta} = \frac{-0.1}{6.04} = -0.017$$

$$G_{c_{lag}} = \frac{s+0.1}{s+0.017}$$

$$GH(s) = \frac{30\,(s+0.1)(s+1.2)}{s(s+0.17)(s+1)(s+4)(s+4.95)}$$



Check e_{ss}

$$\lim_{s\to 0} s\,GH(s) = \frac{30\,s\,(s+0.1)(s+1.2)}{s(s+0.17)(s+1)(s+4)(s+4.95)}$$

$$e_{ss} = 0.09$$